Exercises to *1.6 Logical inference and logical equivalence*

(1) *Determine whether the following inference schemes are logically valid:*

1)
$$p \lor q$$
$$\underline{q}$$
$$p$$

2)
$$\underline{p \to q}$$
$$\neg q \to \neg p$$

3)
$$p \to q$$
$$\underline{q \to r}$$
$$p \to r$$

4)
$$\underline{p \lor (q \leftrightarrow r)}$$
$$(p \lor q) \leftrightarrow (p \lor r)$$

5)
$$p \to q$$
$$\underline{\neg p}$$
$$\neg q$$

6)
$$p \lor \neg q$$
$$\underline{\neg q \leftrightarrow \neg p}$$
$$p \to \neg q$$

7)
$$\underline{p \to (q \to r)}$$
$$(p \to q) \to (p \to r)$$

8)
$$\underline{p \to (q \leftrightarrow r)}$$
$$(p \to q) \leftrightarrow (p \to r)$$

(2) *Determine whether b) is a logical consequence of a):*

1)
- *a*) If Bob didn't do the robbery, Paul did it and Harry lied.
- *b*) Either it isn't true that neither Bob nor Paul did the robbery, or Harry did not lie.

2)
- *a*) The company is responsible if and only if the software is its own and was installed before January.
- *b*) If the software is of the company, then it was installed before January and the company is responsible. If the software is not of the company, then it was not installed before January and the company is not responsible.

3)
- *a*) The ships went through either the southern or the northern strait. If the weather was good, they went through the southern strait and were not attacked by pirates. If they went through the northern strait, they were attacked by pirates.
- *b*) If the ships were not attacked by pirates, the weather was good.

4)
- *a*) In case the sunset is red, if it is windy tonight, the sea will be stormy tomorrow.
- *b*) If the sunset is red or it is windy tonight, the sea will be stormy tomorrow.

5)

   a) If John has a beard, I will either not recognize him or, if I recognize him, I won't greet him.

   b) If I greet John, I will have recognized him, and if I don't greet him, he will have a beard.

6)

   a) If you moralize to others and you are convinced that you are moral, you are not moral. If you are moral, you do not moralize to others and you are not convinced that you are moral.

   b) If you are not convinced that you are moral, you are not moral if and only if you moralize to others.

(3) *Prove the following logical equivalences:*

1) $\neg\neg p \Leftrightarrow p$

2) $p \to q \Leftrightarrow \neg q \to \neg p$

3) $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

4) $p \leftrightarrow q \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$

5) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

6) $p \to q \Leftrightarrow \neg(p \wedge \neg q)$

7) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

8) $[(p \to q) \to p] \Leftrightarrow p$

9) $\neg(p \leftrightarrow q) \Leftrightarrow p \leftrightarrow \neg q$

10) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

(4) *Determine if a) and b) are logically equivalent:*

1)

   a) It's not true that if Alice doesn't call Peter, he'll call her.

   b) It's not true that if Peter doesn't call Alice, she'll call him.

2)

   a) If it is true that if wolfs came at night, the dogs barked, then no wolfs came at night.

   b) Neither wolfs came at night nor the dogs barked.

3)

   a) In case the sunset is red, if it is windy tonight, the sea will be stormy tomorrow.

   b) If the sunset is red and it is windy tonight, the sea will be stormy tomorrow.

4)

   a) If John is guilty, then Peter is innocent, and George is lying.

   b) If George is not lying, then John is not guilty, and if John is guilty, then Peter is innocent.

5)

   a) Kuwait or Saudi Arabia, but not both, will cut oil prices.

   b) Kuwait will cut oil prices if and only if Saudi Arabia does not cut them.

6)

   a) If the guard shot the thief, then if he shot him without warning, the guard is guilty.

   b) The guard is either guilty or warned the thief or did not shoot him.

7)

   a) If Bob have taken Alice's bike without telling her, then he stole it.

   b) If Bob didn't stole Alice's bike, he either hasn't taken it or has told her that he had taken it.

8)

   a) If the swamp is drained and a road is built, in a year people's well-being will improve and the population in the settlement will increase.

   b) If the swamp is drained or a road is built, in a year the population in the settlement will increase. If, in a year, people's well-being is not improved, neither a road will be built nor the swamp will be drained.

9)

   a) I have never told you such a thing, nor have I ever thought of it, nor will I ever think of such a thing, let alone tell you.

   b) Neither if I have never told you such a thing, I will ever tell you that, nor if I have never thought of such a thing, I will ever think of it.