

Exercises to 3.2 Variables and quantifiers

(1) For each of the following formulas, specify:

- the scope of each quantifier
- which variable occurrences are bound and which are free
- whether it symbolizes an open sentence or a statement

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| 1) $\exists x(Fxy \wedge Gx)$ | 6) $\forall y\neg\exists xFxy$ |
| 2) $\exists xFxy \wedge Gx$ | 7) $\neg\forall x(\exists yFxy \wedge Gx)$ |
| 3) $\exists x\forall yFxy \wedge Gx$ | 8) $\forall y[\forall xFxy \rightarrow \neg Gy]$ |
| 4) $\exists x(Fxy \wedge \forall yGyx)$ | 9) $\neg Gx \rightarrow [\neg\forall y(\neg Fxy \vee \exists xGx) \rightarrow Hy]$ |
| 5) $\exists xFxx \vee \neg Gx$ | 10) $\forall y(\exists yFxy \leftrightarrow Gy)$ |

(2) Symbolize the following sentences using the given notation.

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| 1) All bats are mammals. | F - ...is a bat, G - ...is a mammal |
| 2) Nothing is different from itself. | F - ...is different from... |
| 3) Points are not extended. | F - ...is a point, G - ...is extended |
| 4) Some metals are liquid. | F - ...is metal, G - ...is liquid |
| 5) Everything is finite or infinite. | F - ...is finite |
| 6) Everything is finite or everything is infinite. | (as above) |
| 7) Some sharks are not predators. | F - ...is a shark, G - ...is a predator |
| 8) Everyone who likes Socrates likes Plato. | F - ...likes..., a - Socrates, b - Plato, D - the set of all humans |
| 9) If all humans are sinful, then the Pope is sinful too. | F - ...is human, G - ...is sinful, a - the Pope |
| 10) There are insects with 8 legs. | F - ...is an insect, G - ...has 8 legs |
| 11) Bob gave something to Alice. | F - ...gave...to..., a - Alice, b - Bob |
| 12) Alice received nothing from Bob. | (as above) |
| 13) Bob gave Alice a book. | F - ...gave... ..., G - ...is a book, a - Alice, b - Bob |
| 14) Alice has a new bike. | F - ...has..., G - ...is new, H - ...is a bike, a - Alice |
| 15) The otter is not suitable for a pet. | F - ...is an otter, G - ...is suitable for a pet |
| 16) There are no cats that do not purr. | F - ...is a cat, G - ...purrs |

Exercises to 3.2 *Variables and quantifiers*

- 17) I hate snakes. F – ...is a snake, G – ...hates..., a – I
- 18) Only humans laugh. F – ...is human, G – ...laughs
- 19) A necessary condition for something to be an animal is to move. F – ...is an animal, G – ...moves
- 20) A sufficient but not necessary condition for a number to be even is to be divisible by 8. F – ...is even, G – ...is divisible by 8, D – the set of all numbers
- 21) John is not interested in anything but his dogs. F – ...is interested in..., G – ...is a dog, H – ...belongs to..., a – John
- 22) Alice avoids everything she doesn't like. F – ...avoids..., G – ...likes..., a – Alice
- 23) Alice likes everything Bob likes. F – ...likes..., a – Alice, b – Bob
- 24) John cannot outrun everyone in the team. F – ...can outrun..., G – ...is in..., a – John, b – the team
- 25) John cannot outrun anyone in the team. (as above)