Exercises to *3.5 Proof procedure*

(1) *Determine whether the following pairs of expressions are logically equivalent. If not, change one of the quantifiers in the first expression (to existential if it is universal, and vice versa) so that they become equivalent.*

1) $\neg\forall y \qquad \exists y\neg$
2) $\neg\exists x\neg \qquad \neg\neg\forall x$
3) $\forall x\neg\forall y\forall z\neg \qquad \neg\exists x\forall y\neg\forall z$
4) $\forall x\neg\neg\exists y\neg \qquad \neg\exists x\forall y$
5) $\forall x\neg\forall y\forall z\neg \qquad \forall x\exists y\exists z$

6) $\neg\exists x\forall y\forall z \qquad \forall x\exists y\forall z\neg$

7) $\exists x\exists y\neg\forall z\forall w\neg \qquad \exists x\exists y\exists z\exists w$
8) $\exists x\exists y\neg\forall z\forall w\neg \qquad \neg\forall x\forall y\forall z\neg\forall w$
9) $\exists x\exists y\neg\forall z\forall w\neg \qquad \exists x\exists y\exists z\neg\exists w\neg$
10) $\exists x\exists y\forall z\forall w\neg \qquad \neg\forall x\forall y\forall z\exists w$

(2) *Is the instantiation correct? If not, what is wrong?*

1) $$\frac{\forall x(Fxa \to Gbx)}{Fba \to Gbb}$$

2) $$\frac{\forall z(Faz \wedge \neg Fzz) \to Fab}{(Fab \wedge \neg Fbb) \to Fab}$$

3) $$\frac{\exists x\exists y\exists z[(Fxa \wedge \neg Fya) \to Gxyz]}{\exists y\exists z[(Fba \wedge \neg Fya) \to Gxyz]}$$

4) $$\frac{\forall x\exists y[\forall zFzxy \to \exists zGaz]}{\exists y[\forall zFzay \to \exists zGaz]}$$

5) $$\frac{\forall y(Gay \vee \neg\forall xHbxy)}{Gab \vee \neg\forall xHbxa}$$

6) $$\frac{\neg\forall x(Fxa \leftrightarrow Gax)}{\neg(Fba \leftrightarrow Gab)}$$

7) $$\frac{\exists x\forall yFxy \to \forall y\exists xFxy}{\forall yFay \to \forall yFay}$$

8) $$\frac{\forall x[Fxbx \wedge (\neg\exists yGxy \leftrightarrow Hxbx)]}{Fbbb \wedge (\neg\exists yGby \leftrightarrow Haba)}$$

(3) *Below are parts of proofs in which instantiations are used. Are the instantiations correct? If not, what is wrong?*

1)
1. $\neg\forall xFxx$
2. $\exists zGzz$
3. Gaa 2, *EI*
4. $\neg Faa$ 1, *UI*

2)
1. $\forall xFxa$
2. $\exists x\neg Gxx$
3. $\neg Gaa$ 2, *EI*
4. Faa 1, *UI*

3)
1. $\exists xFxb$
2. $\forall yGay$
3. Fbb 1, *EI*
4. Gab 2, *UI*

4)
1. $\forall xFax$
2. $\exists x\neg Fax$
3. Faa 1, *UI*
4. $\neg Fab$ 2, *EI*

5)
1. $\neg\forall xFax$
2. $\exists zGbz$
3. Gba 2, *EI*
4. $\neg Faa$ 1, *UI*

6)
1. $\exists x\exists yFxy$
2. $\forall x\exists yGyx$
3. $\exists yFay$ 1, *EI*
4. $\exists yGya$ 2, *UI*

(4) *Prove (by means of predicate logic) the validity of the following syllogisms:*

1) EAE-1
2) AEE-2
3) OAO-3
4) EIO-2
5) EIO-4
6) AOO-2

(5) *Prove that the inferences are valid:*

1)
$$\frac{\forall x(\neg Fx \rightarrow \neg Gx) \quad \forall xGx}{\exists xFx}$$

2)
$$\frac{\forall x(Fx \wedge Gx)}{\forall xFx \vee \forall xGx}$$

3)
$$\frac{\exists y\forall xFyx}{\forall x\exists yFyx}$$

4)
$$\frac{\forall x(Fx \vee Gx) \quad \forall x(Fx \vee \neg Gx)}{\forall xFx}$$

5)
$$\frac{\exists x(Fx \wedge Gx) \vee \exists x(Fx \wedge \neg Gx)}{\exists xFx}$$

6)
$$\frac{\forall x(Fx \rightarrow \forall y\exists zGxyz) \quad \exists x\forall y\neg Gaxy}{\neg Fa}$$

(6) *Prove the validity of the following inferences using the notations:*

1) No man is perfect.
Socrates is a man.
―――――――――――
Socrates is not perfect.

F – …is a man, *G* – …is perfect, *a* – Socrates

2) Socrates is a philosopher.
Socrates is wise.
―――――――――――
Some philosophers are wise.

F – …is a philosopher, *G* – …is wise, *a* – Socrates

3) Paul or John is not polite.
If John is polite, everyone is polite.
―――――――――――
John is not polite.

F – …is polite, *a* – John, *b* – Paul, *D* – the set of humans

4) John can beat anyone on the team who is older than him.
Paul is not older than any of the team who can beat him.
―――――――――――
Paul is not older than John.

F – …can beat…, *G* – …is older than…, *a* – John, *b* – Paul, *D* – the team members

5) All bodies attract each other.
a and *b* are bodies.
―――――――――――
b attracts *a*.

F – …attracts…, *G* – …is a body

6) Each thing either exists in space and time or it is abstract.
Numbers do not exist in space and time.
———
Numbers are abstract.

F – …is abstract, *G* – …is a number, *H* – …exists in space and time

7) Everyone who has read both *Hamlet* and *Othello* likes Hamlet.
Some who have read *Hamlet* do not like it.
———
Some who have read *Hamlet* have not read *Othello*.

F – …has read *Hamlet*, *G* – …has read *Othello*, *H* – …likes *Hamlet*, *D* – the set of humans

8) No vertebrates that have kidneys are amphibians.
If an amphibian has kidneys, it is a vertebrate.
———
No amphibians has kidneys.

F – …is a vertebrate, *G* – …has kidneys, *H* – …is an amphibian, *D* – animals

9) Only adults who have bought a ticket are allowed in the cinema.
Some who bought a ticket are not adults.
———
Some who are not allowed in the cinema have bought a ticket.

F – …is allowed in the cinema, *G* – …is an adult, *H* – …has bought a ticket, *D* – the set of humans

10) All circles are figures.
———
Everyone who draws a circle draws a figure.

F – …is a circle, *G* – …is a figure, *H* – …draws…

11) There are actions that all people condemn.
———
Everyone condemns one action or another.

F – …is an action, *G* – …is a human, *H* – …condemns…

12) Everyone on the forum likes *Amarcord*.
———
Either no one on the forum has watched Amarcord or someone has watched it and likes it.

F – …is on the forum, *G* – …likes *Amarcord*, *H* – …has watched *Amarcord*, *D* – the set of humans

13) There are actions that are condemned by anyone who condemns any actions.
Everyone condemns one action or another.
———
There are actions that everyone condemns.

F – …is an action, *G* – …is a human, *H* – …condemns…

14) I like anyone who laughs at himself.
I hate anyone who laughs at all his friends.
If I hate someone, I don't like him.
———
If there is someone who laughs at all his friends, then there is someone who is not a friend of himself.

F – I like…, *G* – …laughs at…, *H* – I hate…, *I* – …is a friend of…, *D* – the set of humans

Exercises to *3.5 Proof procedure*

(7) *Prove that the formulas are logically valid.*

1) $\forall x(\forall yFy \to Fx)$
2) $\exists x(\forall yFy \to Fx)$
3) $\forall x(Fx \to \exists yFy)$
4) $\forall x\exists y(\forall zFyzx \to Fyyx)$

(8) *Using the distribution of quantifiers over conjunction or disjunction, determine whether the formulas are logically equivalent.*

1)	$\forall x(\neg Fxa \lor Gxb)$	$\forall x\neg Fxa \lor \forall xGxb$
2)	$\forall x(Fxa \land \neg Gax)$	$\forall xFxa \land \neg\exists xGax$
3)	$\exists x(\neg Fxb \lor \neg Gxa)$	$\neg\forall xFbx \land \neg\forall xGxz$
4)	$\exists y(\neg Gyb \land \neg Hay)$	$\neg\forall yGyb \land \exists y\neg Hay$

(9) *What are the formal properties of the following relations?*

1) “less than or equal” in the set of natural numbers
2) “sister” in the set of humans
3) “neighbor” in the set of humans
4) „the same age as“ in the set of humans
5) “teacher” in the set of humans
6) “mother” in the set of humans
7) “greater by two” in the set of natural numbers
8) “likes” in the set of humans
9) “subordinate” in the army
10) “richer” in the set of humans

(10) *Prove that:*

1) If any relation is *asymmetric*, it is also *irreflexive*.
2) If any relation is *intransitive*, it is also *irreflexive*.