Exercises to *3.7 Logical transformations in predicate logic*

(1) *Prove by transformations that the following pairs of formulas are logically equivalent.*

1) $\forall x(\neg Fx \wedge Gbx)$ $\neg\exists xFx \wedge \forall xGbx$

2) $\forall x[\exists yFyx \to \exists zGzx]$ $\forall x\forall y\exists z(Fyx \to Gzx)$

3) $\neg\forall z(Hz \wedge \exists xFxz) \to \forall yGy$ $\forall y[(\forall zHz \wedge \forall z\exists xFxz) \vee Gy]$

4) $\exists y(Fy \to \forall xGxy)$ $\neg\forall yFy \vee \exists y\forall xGxy$

5) $\exists z\neg(Hzz \wedge Gza)$ $\neg\forall zHzz \vee \exists z\neg Gza$

6) $\forall y[\neg Fay \wedge \forall zGyz] \vee \exists zHz$ $\forall z\neg Hz \to [\neg\exists yFay \wedge \forall y\forall zGyz]$

7) $\forall xFxy \leftrightarrow \neg\exists zGz$ $\exists x(Fxy \to \forall z\neg Gz) \wedge \forall x\exists z(\neg Gz \to Fxy)$

8) $\forall y[Fy \to Hay] \to \exists zHza$ $\exists z\exists y[(Fy \wedge \neg Hay) \vee Hza]$

9) $\exists y[\exists x(Hy \vee \neg Gxy) \to \neg\exists zFz]$ $\forall z[\forall y\exists x(Gxy \to Hy) \to \neg Fz]$

10) $\forall yFy \to \{[\exists xHxx \vee \exists xGx] \to \exists zHzz\}$ $\exists z\exists y\{Fy \to \forall x[(Hxx \vee Gx) \to Hzz]\}$

11) $\forall y[\forall xFx \to (\exists zGzy \wedge Hy)]$ $\exists x[Fx \to \forall y\exists z(Gzy \wedge Hy)]$

12) $\forall x(Hx \wedge Fxy) \to \exists z(Gx \wedge Hz)$ $\exists z[\neg\forall xHx \vee \neg\forall xFxy \vee (Gx \wedge Hz)]$

(2) *Convert each of the following formulas to prenex form.*

1) $\forall x\exists yGxy \to [\exists y\forall zGyz \to \forall x\forall zFxz]$

2) $\exists x\forall yFxy \to [\forall y\exists xGyx \wedge \neg\forall x\exists zHxz]$

3) $[\forall xFx \leftrightarrow \forall xGx] \to \forall xHx$

4) $\exists xFx \to [\exists xGx \leftrightarrow \exists xHx]$